AP Physics – Applying Forces

This section of your text will be very tedious, very tedious indeed. (The Physics Kahuna is just as sorry as he can be.) It's mostly just a bunch of complicated problems and how to solve them. Don't be discouraged by how dry it is – sometimes things that are useful don't come in exciting packages. Life cannot always be MTV and video game quality. This section is actually one of the most important parts of the course.

Key Concept: Of enormous importance in solving kinematic problems is this concept.

> ***The sum of the forces acting on objects at rest or moving with constant velocity is always zero.***

$$\sum \boldsymbol{F} = \mathbf{0}$$

This is a special case of Newton's second law; the special case where the net force acting on the system is zero.

We can further simplify the situation! We can analyze the forces in both the ***x*** and ***y*** directions. For an object in equilibrium (at rest or moving with constant velocity) the sum of the forces in the ***x*** and ***y*** directions must also equal zero.

$$\sum \boldsymbol{F_X} = \mathbf{0} \quad \text{and} \quad \sum \boldsymbol{F_Y} = \mathbf{0}$$

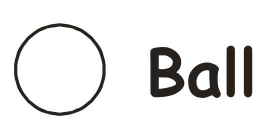
Next Key Concept: Yet another key concept is that when a system is not in equilibrium, the sum of all forces acting must equal the mass times the acceleration that is acting on it, i.e., good old Newton's second law.

$$\sum \boldsymbol{F} = \boldsymbol{ma}$$

Free Body Diagrams: When analyzing forces acting on an object, a most useful thing to do is to draw a ***free body diagram*** or ***FBD***. You draw all the force vectors acting on the system as if they were acting on a single point within the body.

You do not draw the reaction forces.

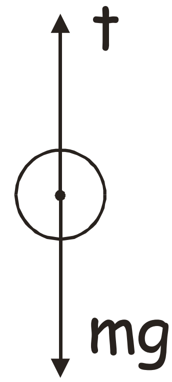
A ball hangs suspended from a string. Let's draw a FBD of the thing. First, draw the ball.



What are the forces acting on it? In this simple case, there are only two forces, the weight of the ball, ***mg***, and the upward force, ***t***, exerted by the string. We call forces that act along strings and chains and such things ***tensions***.

So there are two forces. The weight is directed downward and the tension is directed upward.

Draw the vectors from the center of the ball and label them. You have now made your first free body diagram.



Tension ≡ a pull tangent to a string or rope.

What can we say about the tension and the weight? Well, is the ball moving?

No, it's just hanging. So no motion; that means it is at rest.

What do we know about the sum of the forces acting on it?

If a body is at rest, then the sum of the forces is zero. There are only two forces, the tension and the weight.

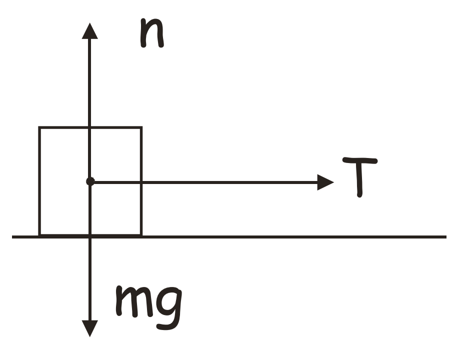
$\sum F = 0$ Therefore:

$t - mg = 0 \quad so \quad t = mg$

Thus, behold! The two forces are equal in magnitude, but opposite in direction (one is up and the other is down).

Problems that involve objects at rest (so the sum of the forces is zero) are called ***static problems***.

Let's look at a typical static problem. We have a crate resting on a frictionless horizontal surface. A force ***T*** is applied to it in the horizontal direction by pulling on a rope - another tension. Let's draw a free body diagram of the system.



There are three forces acting on the crate: the tension from the rope (***T***), the normal force exerted by the surface (***n***), and the weight of the crate (***mg***).

A normal force is a force exerted perpendicular to a surface onto an object that is on the surface.

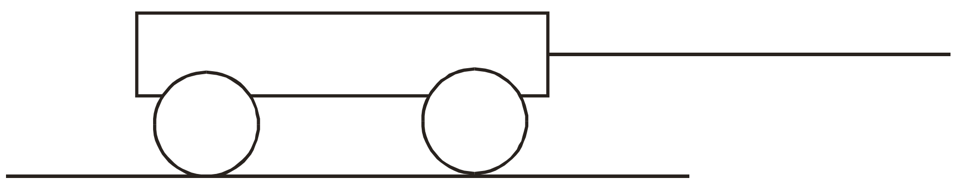
It is important to realize that the normal force is not the reaction force to the object's weight. The reaction force to the object's weight is the force that the object exerts on the earth – recall that the object pulls the earth up just as the earth pulls the object down. The normal force is the table pushing the object up, the reaction force is the object pushing the table down. These action reaction pairs are separate things.

Again, we do not draw the reaction forces on the FBD.

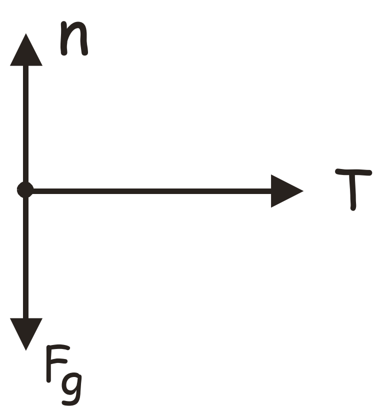
Useful Problem Solving Strategy: Here is a handy set of steps to follow when solving static problems.

1. Make a sketch.
2. Draw a FBD for each object in the system - label all the forces.
3. Resolve forces into ***x*** and ***y*** components.
4. Use $\sum F_X = 0$ and $\sum F_Y = 0$
5. Keep track of the force directions and decide on a coordinate system so you can determine the sign (neg or pos) of the forces.
6. Develop equations using the second law for the ***x*** and ***y*** directions.
7. Solve the equations.

- A crate rests on very low friction wheels. The crate and the wheels and stuff have a weight of 785 N. You pull horizontally on a rope attached to the crate with a force of 135 N. (a) What is the acceleration of the system? (b) How far will it move in 2.00 s?



(a) The forces on the system are: $\boldsymbol{T}$, a tension (the pull on the rope), $\boldsymbol{F_g}$, the weight of the cart, and $\boldsymbol{n}$, the normal force. Let's draw the FBD.



Y direction: There is no motion in the ***y*** direction so the sum of the forces is zero.

$\sum F_y = 0$. This means that the normal force magnitude equals the weight. We can therefore ignore the ***y*** direction.

X direction: The motion in the ***x*** direction is very different. Since there is only one force, the system will undergo an acceleration.

$$\sum F_x = ma$$

Writing out the sum of the forces (only the one), we get:

$$T = ma \qquad a = \frac{T}{m}$$

We need to find the mass; $w = mg$ $\qquad m = \frac{w}{g} = \frac{785 \frac{kg \cdot \cancel{m}}{\cancel{s}^2}}{9.8 \frac{\cancel{m}}{\cancel{s}^2}} = 80.1\,kg$

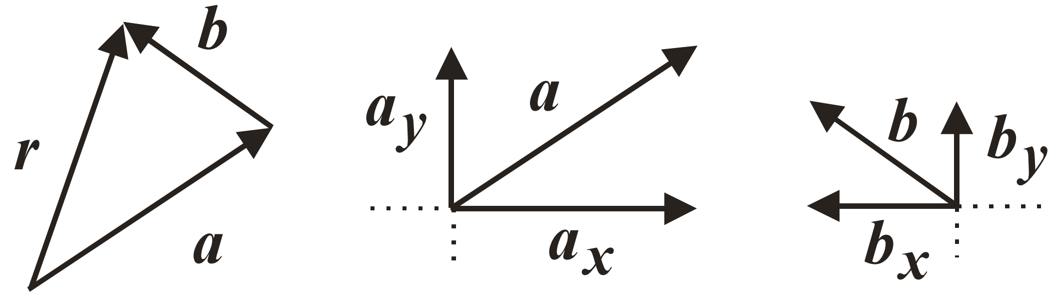
$$a = 135 \frac{\cancel{kg} \cdot m}{s^2}\left(\frac{1}{80.1\,\cancel{kg}}\right) = \boxed{1.69 \frac{m}{s^2}}$$

(b) How far does it travel in 2.00 s?

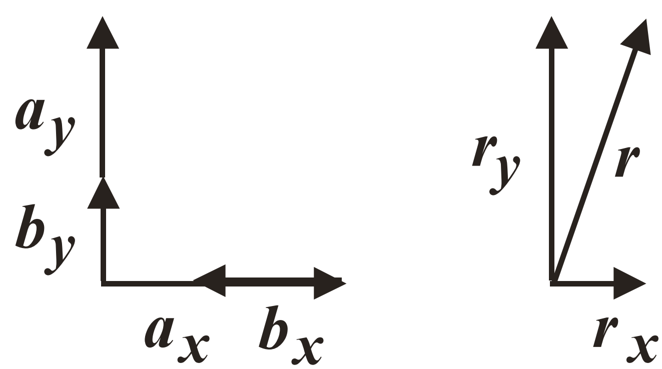
$$x = \frac{1}{2}at^2 = \frac{1}{2}\left(1.69 \frac{m}{\cancel{s}^2}\right)(2.00\,\cancel{s})^2 = \boxed{3.38\,m}$$

Adding Forces: When adding two or more vectors, you find the components of the vectors, then add the components. So you would add the x components together which gives you the resultant x component. Then add the y components obtaining the resultant y component. Then you can find the magnitude and direction of the resultant vector.

You want to add two forces, ***a*** and ***b***. They are shown in the drawing. The resultant force, ***r***, is also shown. To the right you see the component vectors for ***a*** and ***b***.



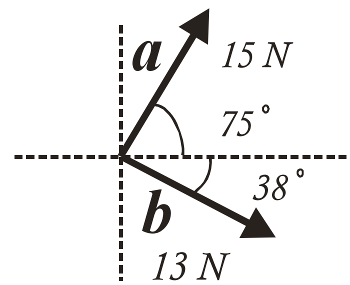
We add the component vectors – it looks like this:



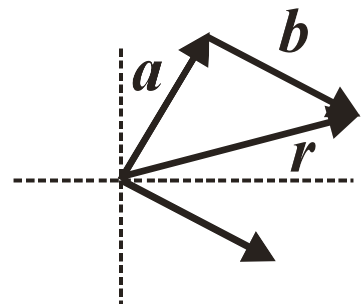
See how you end up with the resultant vector after you've added up the components?

Now let's do a problem where we have to add two forces.

Here is a drawing showing the two vectors:



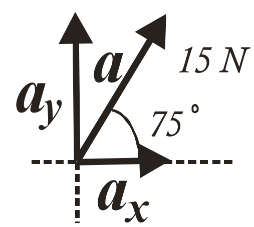
We do a quick sketch showing how the forces add up:



Okay, here's how to add them up.

1. Resolve each vector into its ***x*** and ***y*** components.
2. Add all the ***x*** components to each other and the ***y*** components to each other. This gives you the ***x*** and ***y*** components of the resultant vector.
3. Use the Pythagorean theorem to find the magnitude of the resultant vector.
4. Use the tangent function to find the direction of the resultant.

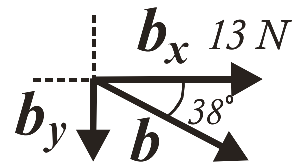
1. Find the ***x*** and ***y*** components for force ***a***:



$$a_x = a\cos\theta \quad = 15\ N\cos 75^{\circ} \quad = \quad 3.88\ N$$

$$a_y = a\sin\theta \quad = 15\ N\sin 75^{\circ} \quad = \quad 14.5\ N$$

2. Find ***x*** and ***y*** components for force ***b***



$$b_x = a\cos\theta \quad = 13\ N\cos 38^{\circ} \quad = \quad 10.2\ N$$

$$b_y = a\sin\theta \quad = 13\ N\sin 38^{\circ} \quad = \quad -8.00\ N$$

2. Add the components:

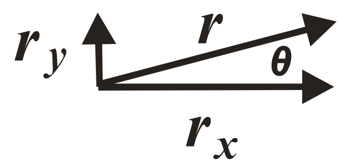
$$r_x = a_x + b_x \quad = 3.88\ N + 10.2\ N \quad = \quad 14.08\ N$$

$$r_y = a_y + b_y \quad = 14.5\ N + (-8.00\ N) \quad = \quad 6.50\ N$$

3. Find the magnitude of the resultant vector (which we shall call r):

$$r^2 = r_x^{\ 2} + r_y^{\ 2} \quad r = \sqrt{r_x^{\ 2} + r_y^{\ 2}} \quad = \sqrt{(14.08\ N)^2 + (6.50\ N)^2} \quad = \quad \boxed{15.5\ N}$$

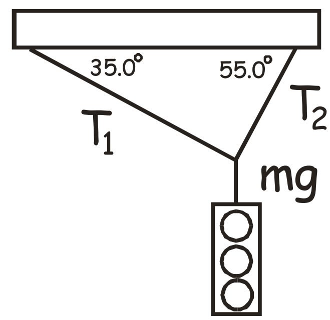
4. Find the direction of the resultant force:



$$\theta = \tan^{-1}\left(\frac{r_Y}{r_X}\right)$$

$$\theta = \tan^{-1}\left(\frac{6.50\ \cancel{N}}{14.08\ \cancel{N}}\right) = \boxed{24.8^{o}}$$

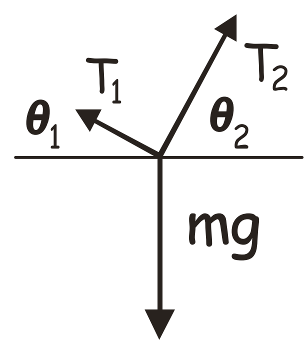
- A 85.0 kg traffic light is supported as shown. Find the tension in each cable.



There are three forces acting on the traffic light, $\boldsymbol{T_1}$, $\boldsymbol{T_2}$, and $\boldsymbol{mg}$. $\boldsymbol{T_1}$ is the tension from the left cable, $\boldsymbol{T_2}$ is the tension from the right cable, and $\boldsymbol{mg}$ is the weight of the light.

To solve the problem, we must resolve the two tensions into their $\boldsymbol{x}$ and $\boldsymbol{y}$ components and then add up the forces in the $\boldsymbol{x}$ and $\boldsymbol{y}$ directions. We know that the sum of these forces must equal zero.

Here is the FBD for the problem: We can identify the two angles by using a little geometry. $\boldsymbol{\theta_1}$ is 35.0° and $\boldsymbol{\theta_2}$ is 55.0°.



Let's look at the forces acting in the $\boldsymbol{x}$ direction.

$$\sum F_X = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

The only forces acting in the $\boldsymbol{x}$ direction are the two $\boldsymbol{x}$ components from the tension. The weight has no $\boldsymbol{x}$ component since its direction is straight down. The two $\boldsymbol{x}$ component forces are in opposite directions.

Now we can write out an equation for the sum of the forces in the y direction.

$$\sum F_y = T_2 \sin\theta_2 + T_1 \sin\theta_1 - mg = 0$$

We've let down be negative and up be positive.

We now have two equations with two unknowns, so we can solve the equations simultaneously.

Solve for $\boldsymbol{T_1}$ in the first equation:

$$\sum F_X = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \qquad T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

$$T_1 = T_2 \frac{\cos\theta_2}{\cos\theta_1} = T_2\left(\frac{\cos 55.0^{o}}{\cos 35.0^{o}}\right) = 0.7002\, T_2$$

Plug this value into the second equation:

$$T_2 \sin\theta_2 + (0.7002\, T_2)\sin\theta_1 - mg = 0 \quad T_2\left(\sin\theta_2 + 0.7002 \sin\theta_1\right) = mg$$

$$T_2 = \frac{mg}{\left(\sin\theta_2 + 0.7002 \sin\theta_1\right)}$$

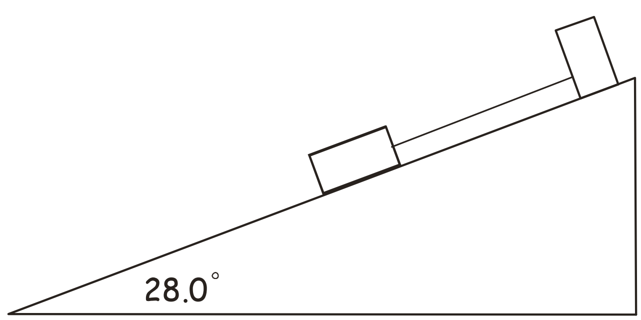
$$T_2 = \frac{85.0\, kg\left(9.8 \frac{m}{s^2}\right)}{\left(\sin 55.0^\circ + 0.7002 \sin 35.0^\circ\right)} = \boxed{682\ N}$$

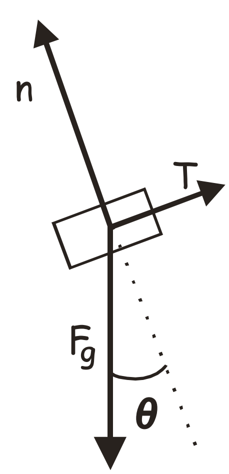
Now we can find ***T₁*** by plugging the value of ***T₂*** into the first equation, which we already solved for ***T₁***.

$$T_1 = 0.7002\, T_2 \quad = 0.7002(682\ N) = \boxed{478\ N}$$

Lovely Ramp Problems: A common type of kinematic problem involves an object at rest upon or moving along the surface of an elevated ramp.

Here's a simple problem. A frictionless ramp is elevated at a 28.0° angle. A block rests on the surface and is kept from sliding down by a rope tied to a secure block as shown





If the block has a weight of 225 N, what is the force on the rope holding it up?

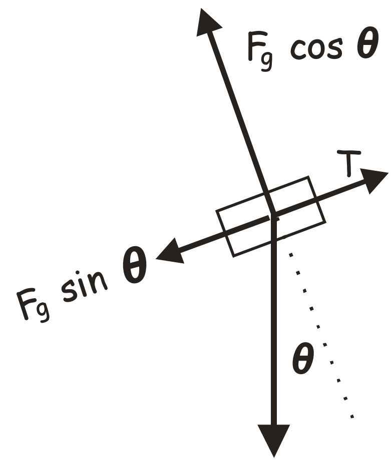
First, let's draw a FBD:

Next we have to choose ***x*** and ***y*** coordinates:

The positive ***x*** direction is up the surface of the ramp – parallel to the surface.

The positive ***y*** direction is perpendicular to the surface of the ramp.

There is a component of the block's weight, F_g, that is directed down the surface of the ramp, which would be along the ***x*** axis. This force is $F_g \sin \theta$. The normal force will be $F_g \cos \theta$.



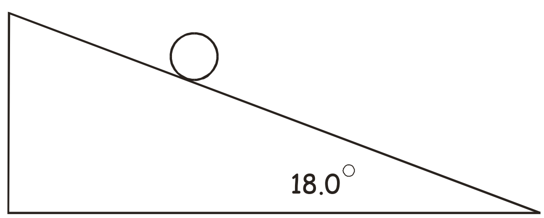
$\sum F_x = 0$ and $\sum F_y = 0$

x direction: ***T*** is balanced by a force down the ramp

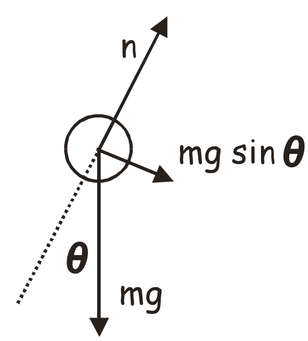
$$T - F_g \sin\theta = 0 \qquad T = F_g \sin\theta$$

$$T = (225\ N)\sin 28^o \quad = \quad \boxed{106\ N}$$

- A 5.00 kg ball slides down a 18.0° ramp. (a) What is the acceleration of the ball? Ignore friction. (b) If the ramp is 2.00 m long, how much time to reach the bottom?



Here's the FBD:



We'll let the direction down the ramp be the positive ***x*** direction; the ***y*** direction will be perpendicular to the surface of the ramp.

(a) First we look at the sum of the forces in the ***x*** direction (up and down the ramp).

$$\sum F_x = mg \sin \theta = ma$$

There is only one force acting in this direction, the component of the weight that is down the slope:

$$mg\sin\theta = ma \quad a = g\sin\theta \quad = \left(9.8\frac{m}{s^2}\right)\sin 18^{\circ} \quad = \quad \boxed{3.03\frac{m}{s^2}}$$

(b) Since we know the acceleration and the distance it goes down the ramp, it's a simple matter to calculate the time it takes to do this.

$$x = \frac{1}{2}at^2 \quad t = \sqrt{\frac{2x}{a}} \quad = \sqrt{2(2.00\,\cancel{m})\left(\frac{1}{3.03\frac{\cancel{m}}{s^2}}\right)} \quad = \quad \boxed{1.15\,s}$$

Notice that we pretty much ignored the ***y*** direction. This was because there was no motion in that direction.

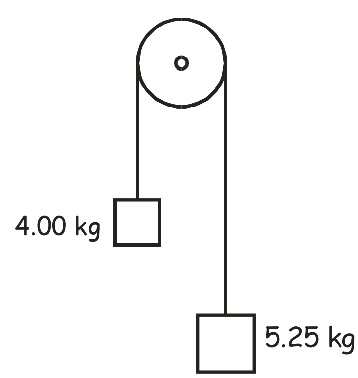
The Origin of the term "Nerd":

The Nerd word has two popular stories toward its origin. One is that it comes from Dr. Seuss's *If I Ran the Zoo*, in which appears a creature called a "nerd." This book was published in 1950. The second is that it is a variation on the name of ventriloquist Edgar Bergen's (Candace's father) dummy, Mortimer Snerd.

Both stories could be correct. There is no cite of the term prior to its 1950 appearance in the Dr. Seuss book. The earliest cite of the current usage is from 1951. Lighter, however, cites a 1941 use of the nickname Mortimer Snerd to refer to a technical, brainy type of guy.

http://www.idiomsite.com/nerd.htm

Two Body Problems: So far we've dealt with only one body. Let's expand the use of Newton's laws to deal with multiple body situations. To solve these problems, each body is treated separately. You draw a FBD for each object an then analyze the forces that are acting. This will give you several equations that can be used to solve the problem.



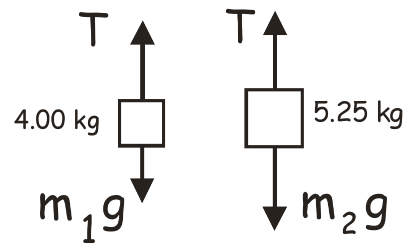
- Two masses, 4.00 kg and 5.25 kg are connected by a light string to a frictionless pulley as shown. Find the tension in the string, and the acceleration on the system.

A single pulley as we have here simply changes the direction of the forces. With the weights arranged as they are, we can see that the heavy weight will move downward and the lighter mass will move up. We will treat the as if they are in one dimension, however.

As we have two bodies, we must draw a FBD for each of them.

Each body experiences two forces; the tension in the string (***T***) which has the same magnitude for each of them (although it is directed in opposite directions), and their weight (***m_1g*** and ***m_2g***).

Here are the FBD's for each:



For the forces on the rising mass, we use up as the positive direction:

$$\sum F_y = m_1 a \qquad T - m_1 g = m_1 a$$

For the falling mass, down is positive

$$\sum F_y = m_2 a \qquad m_2 g - T = m_2 a$$

Note that the acceleration on both masses ***is the same***.

Add the 2 equations:

$$m_1 a = T - m_1 g \quad + \quad m_2 a = m_2 g - T$$

$$m_1 a + m_2 a = \cancel{T} - m_1 g + m_2 g - \cancel{T}$$

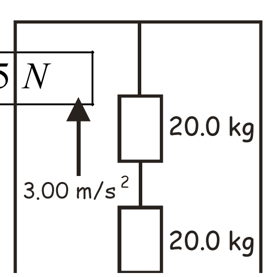
$$a(m_1 + m_2) = m_2 g - m_1 g \qquad a = g\frac{(m_2 - m_1)}{(m_1 + m_2)}$$

$$a = \left(9.80\frac{m}{s^2}\right)\frac{(5.25\ \cancel{kg} - 4.00\ \cancel{kg})}{(5.25\ \cancel{kg} + 4.00\ \cancel{kg})} = \left(9.80\frac{m}{s^2}\right)\left(\frac{1.25}{9.25}\right) = 1.32\frac{m}{s^2}$$

We've solved for the acceleration, so we can use that to find the tension:

$$m_1 a = T - m_1 g \qquad T = m_1 a + m_1 g$$

$$T = (4.00\ kg)\left(1.32\frac{m}{s^2}\right) + (4.00\ kg)\left(9.80\frac{m}{s^2}\right) = \boxed{44.5\ N}$$



- 2 blocks hang in an elevator as shown. The elevator accelerates upward at 3.00 m/s^2. Find the tension in each rope.

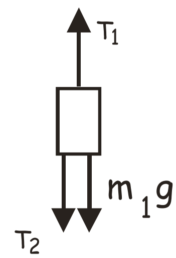
It is important to realize that both blocks will experience the same accelerations as the elevator, 3.00 m/s^2. We will also treat each object separately.

Look at the forces on the upper block in a skillfully drawn FBD:

There is: T_1 up

T_2 down

m_1g down



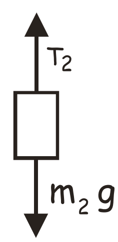
We sum these forces:

$$T_1 - T_2 - m_1 g = m_1 a$$

We can't solve anything here because we have too many unknowns – the two tensions to be specific.

Let us now look upon the lower block:

$$T_2 - m_2 g = m_2 a$$



This we can solve as there is only one unknown.

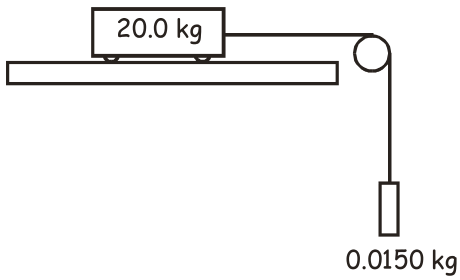
$$T_2 = m_2 g + m_2 a \qquad T_2 = m_2\left(g + a\right) \quad = 20.0\ kg(9.8\frac{m}{s^2} + 3.0\frac{m}{s^2}) \quad = \quad \boxed{256\ N}$$

Now we can find the tension on upper block:

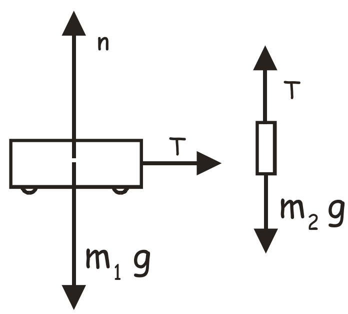
$$T_1 - T_2 - m_1 g = m_1 a \qquad T_1 = T_2 + m_1 g + m_1 a \qquad = T_2 + m_1\left(g + a\right)$$

$$T_1 = 256\ N + 20.0\ kg\ (9.8\frac{m}{s^2} + 3.0\frac{m}{s^2}) \quad = \quad \boxed{512\ N}$$

- A 20.0 kg cart with very low friction wheels sits on a table. A light string is attached to it and runs over a low friction pulley to a 0.0150 kg mass. What is the acceleration experienced by the cart?



FBD's:



We shall choose the direction of motion to be positive. So for the cart, positive is to the right and for the weight positive will be down. Okay?

Sum the forces on each object:

Cart: $T = m_1 a$ Note that there is only one force acting on the cart in the horizontal direction.

Hanging mass: $m_2 g - T = m_2 a$ Two forces act on the hanging mass.

Both bodies experience the same acceleration.

We can add the two equations together.

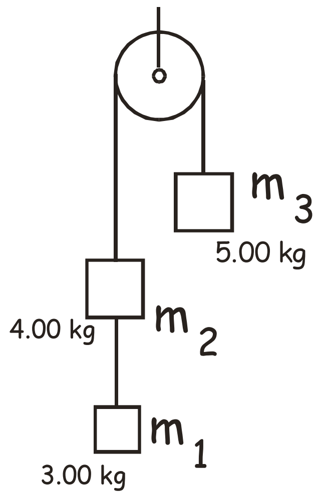
$$T = m_1 a \quad + \quad m_2 g - T = m_2 a \qquad \cancel{T} + m_2 g - \cancel{T} = m_1 a + m_2 a \qquad m_2 g = m_1 a + m_2 a$$

Note how the unknown tension canceled out, leaving us with a single equation with only one unknown, a thing we can now solve.

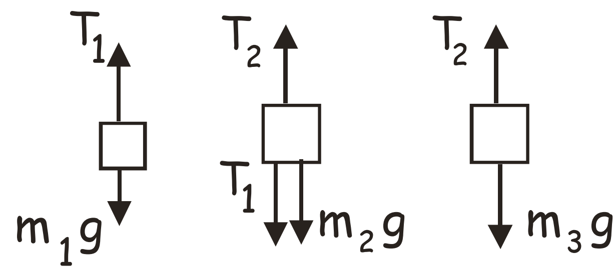
$$a\left(m_1 + m_2\right) = m_2 g \qquad a = \frac{m_2 g}{m_1 + m_2}$$

$$a = 0.0150\,\cancel{kg}\left(9.8\frac{m}{s^2}\right)\left(\frac{1}{20.0\,\cancel{kg} + 0.0150\,\cancel{kg}}\right) = \boxed{0.00734\,\frac{m}{s^2}}$$

- 3 masses hang as shown, they are connected by light strings and your basic frictionless pulley. (a) Find the acceleration of each mass and (b) the tensions in the 2 strings.



FBD's:



Key point: the magnitude for the acceleration for each mass is the same.

For falling masses (left side) down is positive. Up is positive on the upper mass.

m_1 : $\quad m_1 g - T_1 = m_1 a$

m_2 : $\quad m_2 g + T_1 - T_2 = m_2 a$

m_3 : $\quad T_2 - m_3 g = m_3 a$

Add the equations and solve for the acceleration:

$$m_1 g + m_2 g - m_3 g = m_1 a + m_2 a + m_3 a$$

$$g\left(m_1 + m_2 - m_3\right) = a\left(m_1 + m_2 + m_3\right)$$

$$a = \frac{g\left(m_1 + m_2 - m_3\right)}{\left(m_1 + m_2 + m_3\right)}$$

$$a = \left(9.8\frac{m}{s^2}\right)\frac{\left(3.00\,\cancel{kg} + 4.00\,\cancel{kg} - 5.00\,\cancel{kg}\right)}{\left(3.00\,\cancel{kg} + 4.00\,\cancel{kg} + 5.00\,\cancel{kg}\right)} = \boxed{1.63\,\frac{m}{s^2}}$$

Find the tensions: $\quad m_1 g - T_1 = m_1 a \qquad m_1 g - m_1 a = T_1$

$$T_1 = m_1\left(g - a\right) \qquad T_1 = 3.00\,kg\left(9.80\frac{m}{s^2} - 1.63\frac{m}{s^2}\right) = \boxed{24.5\,N}$$

$$T_2 - m_3 g = m_3 a \qquad T_2 = m_3\left(g + a\right)$$

$$T_2 = 5.00\,kg\left(9.80\frac{m}{s^2} + 1.63\frac{m}{s^2}\right) = \boxed{57.2\ N}$$

A historical e-mail received by the Physics Kahuna.

A few weeks ago on this list we were discussing weight and a test question on the recent PSAT Test.

6. METER : DISTANCE ::
(a) ounce : pound
(b) gram : weight
(c) container : liquid
(d) size : height
(e) boundary : periphery

My wife wrote a letter to the Educational Testing Service to complain about the question. She wrote:

> "The correct answer is none of the above, but I'm afraid (even saddened and angry) that you will count the correct answer as (b). A gram is not a unit of weight but a unit of mass, something that I strive on a daily basis to get my students to understand. A question like this on the PSAT only demonstrates why US students have so many problems on international tests.
>
> It is true that the answer (b) is the only choice of *units: quantity* but it also reinforces a misconception that is common in our society and creates huge barriers to understanding physics and chemistry."

She received this response:

Dear Ms. DeBruyckere: as one of the assessment specialists responsible for the verbal sections of the PSAT/NMSQT, I am writing in response to your inquiry regarding an analogy question in the test administered on October 12, 1999.

The capitalized terms of the analogy are "METER:DISTANCE"; the relationship between these terms can be stated as "The 1st is a unit that may be used to quantify a 2nd." Of the five choices offered, only (B), "gram:weight," displays a relationship similar to that of the capitalized terms. While it is true, from a strictly scientific point of view, that a gram is a unit of mass and not of weight per se, in common, everyday use grams are often thought of as units of weight.

For example, on the nutrition facts label on a certain can of tuna the serving size is said to be "1 can (2.8oz/78g)"; the weight of a certain chocolate bar is given as "3.5 oz/100g." Merriam Webster's Collegiate Dictionary (Tenth Edition) gives as a second definition of gram "the weight of a gram under standard gravity." In many "real world" situations, ounces and grams, as well as pounds and kilograms, are used interchangeably to quantify weight. This practice is reflected in the standard conversion tables that give the U.S. equivalents of metric units: a gram is said to be equivalent to approximately 0.035 ounce (and ounces, of course, are units of weight). Again, it is indisputable that for a scientist, especially a physicist, a gram is a unit of mass; an object with mass does not have weight until acted on by a gravitational force. The context of the analogy, however, makes it